## The Geometry of Right Angles

When squaring by measurement, always check both the lengths of the sides and the diagonals. Parallel opposite sides, equal opposite sides or equal diagonals alone do not guarantee that the adjacent sides of the two figures below are at right angles to one another.


## Isosceles Trapezoid

Diagonals equal
One pair of opposite sides equal One pair of opposite sides parallel


## Parallelogram

Opposite sides equal Opposite sides parallel


Only a rectangle meets all of the following conditions:
Opposite sides are equal
Opposite sides are parallel
Diagonals are equal
Adjacent sides are at right angles to one another


## Pythagorean Triples

Right triangles with sides consisting of whole numbers can be constructed using the following formulas:
Select any two positive numbers, $\boldsymbol{x}$ and $\boldsymbol{y}$, where $\boldsymbol{x}>\boldsymbol{y}$
Example: Let $\boldsymbol{x}=5$, and $\boldsymbol{y}=3$
Substitute for $\boldsymbol{x}$ and $\boldsymbol{y}$ in the equations:

$$
\begin{array}{ll}
\mathbf{a}=\boldsymbol{x}^{2}-\boldsymbol{y}^{2} & \mathbf{a}=25-9=\mathbf{1 6} \\
\mathbf{b}=2 \boldsymbol{x} \boldsymbol{y} & \mathbf{b}=2 \times 5 \times 3=\mathbf{3 0} \\
\mathbf{c}=\boldsymbol{x}^{2}+\boldsymbol{y}^{2} & \mathbf{c}=25+9=\mathbf{3 4}
\end{array}
$$

The results always conform to the Pythagorean Theorem:

$$
a^{2}+b^{2}=c^{2} \quad 16^{2}+30^{2}=34^{2}=1156 *
$$

Example proportions of $\mathbf{a}: \mathbf{b}: \mathbf{c}$ generated by the formulas:

$$
3: 4: 5
$$

5:12:13
7:24:25
8:15:17
20:21:29
28:45:53
33:56:65
39:80:89
The basic ratios may be scaled to convenient lengths or units by multiplying or dividing all of the terms in the ratio by the same number:
$\{3: 4: 5\} \times \mathbf{1 0 0}=300 \mathrm{~cm}: 400 \mathrm{~cm}: 500 \mathrm{~cm}$ ( metric scale)
$\{8: 15: 17\} \times 2=16: 30: 34 *$ (compare to example)
$\{5: 12: 13\} \times \mathbf{1 2}=60^{\prime \prime}: 144^{\prime \prime}: 156^{\prime \prime}$ (feet to inches)
$\{20: 21: 29\} \div 2=10^{\prime}-0^{\prime \prime}: 10^{\prime}-6^{\prime \prime}: 14^{\prime}-6^{\prime \prime}$

